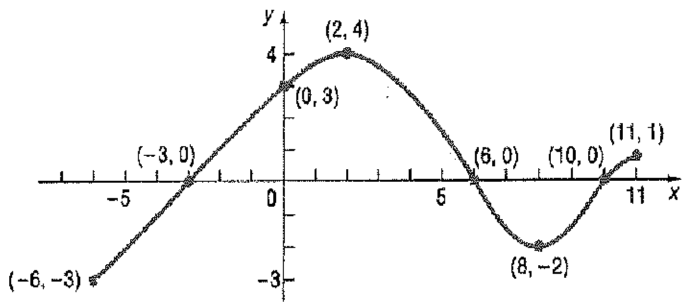
Limits

Limits: Graphical Solutions

Graphical Limits

Let $f(x)$ be a function defined on the interval [-6,11] whose graph is given as:



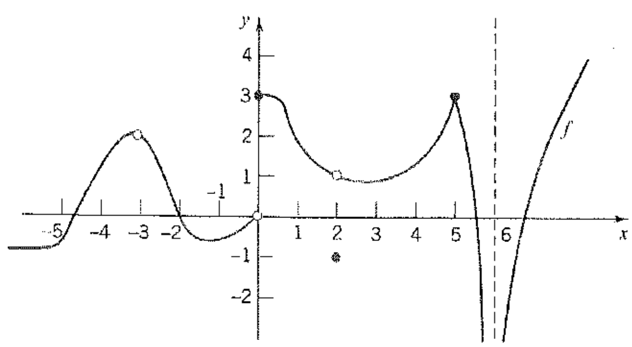
The limits are defined as the value that the function approaches as it goes to an x value. Using this definition, it is possible to find the value of the limits given a graph. A few examples are below:

$$\lim_{x\to -3} f(x) = 0 \qquad \lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 8} f(x) = -2$$

In general, you can see that these limits are equal to the value of the function. This is true if the function is <u>continuous</u>.

Continuity

Continuity of a graph is loosely defined as the ability to draw a graph without having to lift your pencil. To better understand this, see the graph below:



Let's investigate at the flowing points:

$x = -3$ Discontinuous at this point *The value is not defined at -3* "Removable discontinuity"	$x = 0$ Discontinuous at this point *The limit of the left is not equal to the limit from the right* "Jump discontinuity"	$x = 2$ Discontinuous at this point *The limit from the left is equal to the right, but is not equal to the value of the function* "Removable discontinuity"
$x = 4$ Continuous at this point *The limit from the left is equal to the limit from the right and equal to the value of the function*	$x = 5$ Continuous at this point *The limit from the left is equal to the limit from the right and equal to the value of the function*	$x = 6$ Discontinuous at this point *The value of the limit is equal to negative infinity and therefore not defined* "Infinite discontinuity"

One-Sided Limits: General Definition

One-sided limits are differentiated as *right-hand limits* (when the limit approaches from the right) and *left-hand limits* (when the limit approaches from the left) whereas ordinary limits are sometimes referred to as *two-sided limits*. Right-hand limits approach the specified point from positive infinity. Left-hand limits approach this point from negative infinity.

The right-handed limit:

$$\lim_{x \to a^+} f(x) = L$$

The left-handed limit:

$$\lim_{x \to a^-} f(x) = L$$

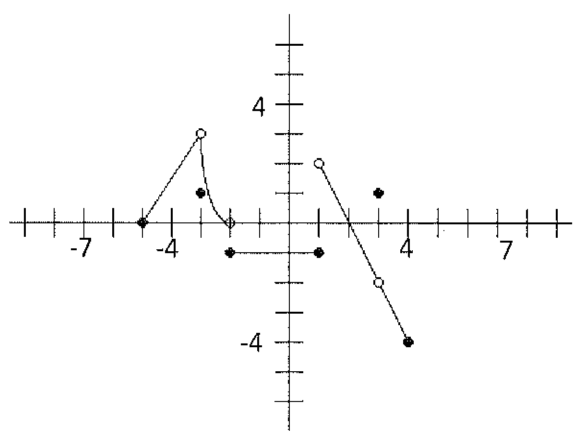
A More Formal Definition of Continuity

From this information, a more formal definition can be found. Continuity, at a point *a*, is defined when the limit of the function from the left equals the limit from the right and this value is also equal to the value of the function. Using notation, for all points *a* where

$$lim_{x\to a^+} f(x) = lim_{x\to a^-} f(x) = f(a),$$

the function is said to be ***continuous***.

A. Now you try some!



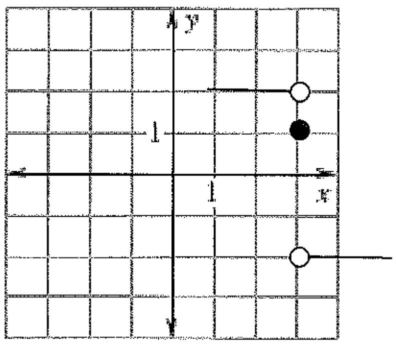
Determine if the following limits exists:

$\lim_{x\to -3} f(x)$ $\lim_{x\to -2} f(x)$ $\lim_{x\to 0} f(x)$ $\lim_{x\to 1} f(x)$ $\lim_{x\to 2} f(x)$ $\lim_{x\to 3} f(x)$

Honors Pre-Calculus **Limits Worksheet** **Name________________________**

Use the graph to estimate the limits and function values, or explain why the limits do not exist or the function values are undefined.

1.



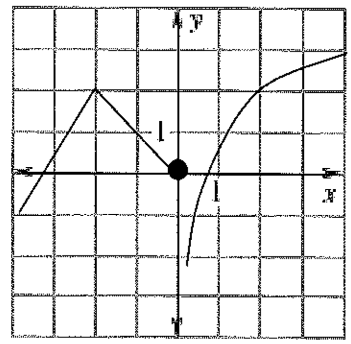
a. $\lim_{x \to 3^-} =$ ____________

b. $\lim_{x \to 3^+} =$ ____________

c. $\lim_{x \to 3} =$ ____________

d. $f(3) =$ ____________

2.



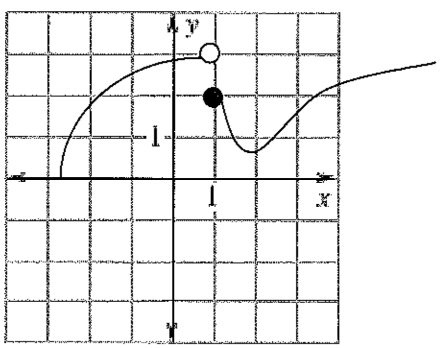
a. $\lim_{x \to -2^-} =$ ____________

b. $\lim_{x \to -2^+} =$ ____________

c. $\lim_{x \to -2} =$ ____________

d. $f(-2) =$ ____________

3.



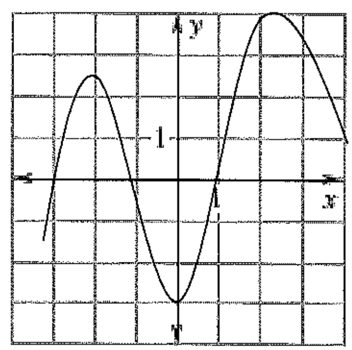
a. $\lim_{x \to 1^-} =$ ____________

b. $\lim_{x \to 1^+} =$ ____________

c. $\lim_{x \to 1} =$ ____________

d. $f(1) =$ ____________

4.



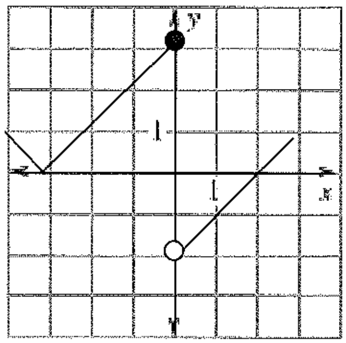
b. $\lim_{x \to 0^-} =$ ____________

c. $\lim_{x \to 0^+} =$ ____________

d. $\lim_{x \to 0} =$ ____________

e. $f(0) =$ ____________

5.



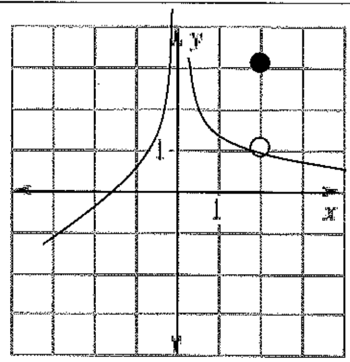
a. $\lim_{x \to 0^-} =$ ____________

b. $\lim_{x \to 0^+} =$ ____________

c. $\lim_{x \to 0} =$ ____________

d. $f(0) =$ ____________

6.



a. $\lim_{x \to 2^-} =$ ____________

b. $\lim_{x \to 2^+} =$ ____________

c. $\lim_{x \to 2} =$ ____________

d. $f(2) =$ ____________

Determine the limit.

7. $\lim_{x \to -\frac{1}{2}} 3x^2(2x - 1)$

8. $\lim_{x \to -4} (x + 3)^{1997}$

9. $\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 3}$

10. $\lim_{x \to 0} e^x \cos x$

11. $\lim_{x \to -2} \sqrt{x - 2}$

12. $\lim_{x \to 0} \frac{1}{x^2}$

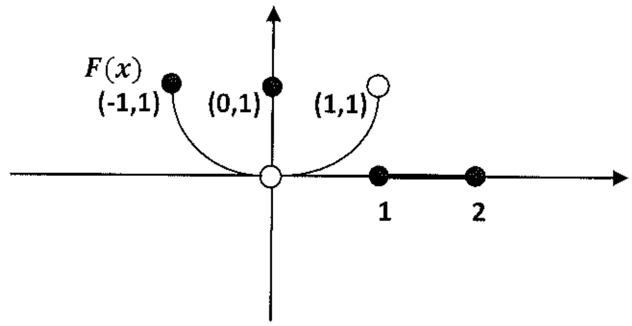
13. $\lim_{x \to 1} \frac{x - 1}{x^2 - 1}$

14. $\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$

15. $\lim_{x \to 0} \frac{(2 + x)^3 - 8}{x}$

16. $\lim_{x \to 0} \frac{\sin 2x}{x}$ *hint: graph this one!

17.



a. $\lim_{x \to 0^-} f(x) =$ ___________

b. $\lim_{x \to 0^+} f(x) =$ ___________

c. $\lim_{x \to 0} \; f(x) =$ ___________

d. $\lim_{x \to 1^-} f(x) =$ ___________

e. $\lim_{x \to 1^+} f(x) =$ ___________

f. $\lim_{x \to 1} \; f(x) =$ ___________

g. $\lim_{x \to 2^-} f(x) =$ ___________

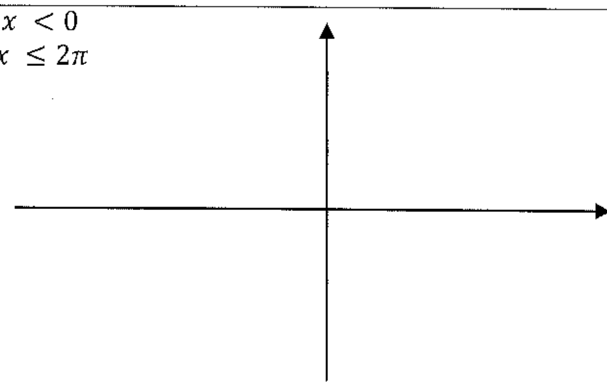
h. $\lim_{x \to 2^+} f(x) =$ ___________

i. $\lim_{x \to 2} f(x) =$ ___________

j. $f(0) =$ ___________

k. On the interval $[-1,1]$, $\mathrm{f(x)}$ is discontinuous at $\mathrm{x} =$ ___________

18. Given the piecewise function $f(x) = \begin{cases} \sin x, & -2\pi \le x < 0 \\ \cos x, & 0 \le x \le 2\pi \end{cases}$



a. Draw the graph

b. At what points does ***only*** the left hand limit exist?

c. At what point does ***only*** the right hand limit exist?